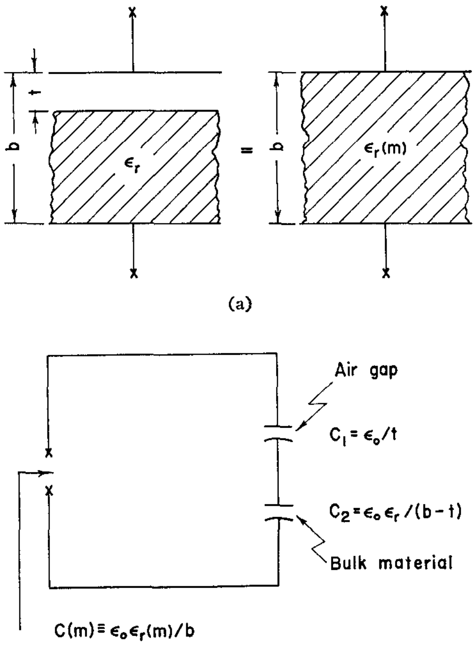

Fig. 3. Physical interpretation of Eq. (2). (a) Parallel plane model. (b) Equivalent circuit.

simplicity results from the fact that (2) considers the effect of the gap on only the dominant mode and neglects all higher-order modes generated at the planes of discontinuity between empty and "filled" waveguides, an approach that is rigorously justified for small perturbations only

$$(|\{\epsilon_r - 1\}\{t/b\}| \ll 1).$$

Since the maximum value of

$$|\{\epsilon_r - 1\}\{t/b\}|$$

was about 13 in the present experiment, one sees that (2) gives a "surprisingly" good qualitative description of "gap effect" for large perturbations. One should generally not expect it to be quantitatively accurate in this range, however.

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Fringing Capacitance in Strip-Line Coupler Design

Very useful relationships between strip-line directional coupler dimensions and the even- and odd-mode impedance have been derived by S. B. Cohn for both the case of side-by-side strips [1] (edge coupling) and broadside coupling [2]. In each case the relations for even- and odd-mode impedance contain, respectively, terms for even- and odd-mode fringing capacitance per unit length. Cohn has derived relationships for both even- and odd-mode fringing capacitances for the case of side-by-side strips [1] and broadside coupling [2] for strips of zero thickness, and has published a paper on thickness corrections [3]. Gunderson and Guida [4] have shown that for the broadside coupled case the even- and odd-mode fringing capacitances are not independent and have thus derived a relationship between the coupler dimensions and even and odd impedance which does not involve expressions for the fringing capacitances. This has led them to formulate a simpler design procedure [4].

The purpose of this communication is to show that such a relation also exists for side-by-side strips.

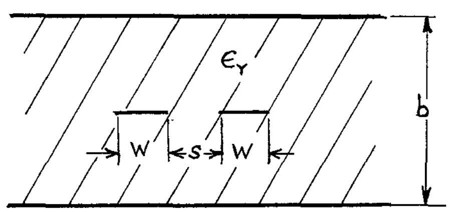

Fig. 1. Side-by-side strip coupler cross section.

Cohn's equations [1] (11) and (12) can be written [5] when $W/b \to 0.35$:

$$Z_1 = \frac{\eta_0}{4\sqrt{\epsilon_r}\left[\frac{W}{b} + \frac{C_f + C_{f1}}{2\epsilon_r\epsilon_0}\right]} \text{ (in } \Omega)$$

$$Z_2 = \frac{\eta_0}{4\sqrt{\epsilon_r}\left[\frac{W}{b} + \frac{C_f + C_{f2}}{2\epsilon_r\epsilon_0}\right]} \text{ (in } \Omega)$$

from which one can solve for

$$C_{f2} - C_{f1} = \frac{\eta_0\epsilon_0\sqrt{\epsilon_r}}{2}\left(\frac{1}{Z_2} - \frac{1}{Z_1}\right) \text{ (in F/m)}$$

where:

Z_1 = odd characteristic impedance
Z_2 = even characteristic impedance
η_0 = intrinsic impedance of free space $= 377\ \Omega$
ϵ_0 = permittivity of free space $= 8.85 \times 10^{-12}$ F/m
ϵ_r = relative permittivity of strip-line dielectric
b = separation between ground planes
W = width of each conducting strip
C_f = fringing capacitance from outside edges of strip-to-ground planes (in F/m)
C_{f1} = fringing capacitance at adjacent edges of strips under odd-mode excitation (in F/m)
C_{f2} = fringing capacitance at adjacent edges of strips under even-mode excitation (in F/m).

Also Cohn's equations [1] (13) and (14) can be solved to give

$$C_{f2} - C_{f1} = \epsilon_r\epsilon_0\frac{2}{\pi}\left[-\ln\left(\cosh\frac{\pi s}{2b}\right) + \ln\left(\sinh\frac{\pi s}{2b}\right)\right]$$

$$C_{f2} - C_{f1} = \epsilon_r\epsilon_0\frac{2}{\pi}\ln\left(\tanh\frac{\pi s}{2b}\right)$$

where:

s = separation between strips.

This can be equated to the relation for fringing capacitances given above to give

$$\frac{s}{b} = \frac{2}{\pi}\operatorname{arctanh}\left[\exp\frac{\eta_0}{4\sqrt{\epsilon_r}}\left(\frac{1}{Z_2} - \frac{1}{Z_1}\right)\right]$$

$$\frac{s}{b} = -\frac{1}{\pi}\ln\left[\tanh\frac{\pi\eta_0}{8\sqrt{\epsilon_r}}\left(\frac{1}{Z_1} - \frac{1}{Z_2}\right)\right].$$

Thus, the ratio of strip separation (between adjacent edges) to ground plane separation can be determined without the need to evaluate fringing capacitance. If desired this ratio can be expressed directly in terms of the midband voltage-coupling ratio k, since [4]

$$Z_1 = Z_0\sqrt{\left(\frac{1-k}{1+k}\right)}$$

where Z_0 = characteristic impedance

$$Z_2 = Z_0\sqrt{\left(\frac{1+k}{1-k}\right)}$$

$$\frac{s}{b} = -\frac{1}{\pi}\ln\left\{\tanh\left[\frac{\pi\eta_0}{4\sqrt{\epsilon_r}Z_0}\frac{k}{\sqrt{1-k^2}}\right]\right\}$$

and [1] for strips of zero thickness

$$\frac{W}{b} = \frac{\eta_0}{4\sqrt{\epsilon_r}Z_0}\sqrt{\left(\frac{1-k}{1+k}\right)} - \frac{s}{2b} + \frac{1}{\pi}\left[\ln\cosh\frac{\pi s}{2b} - \ln 2\right].$$

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